1 WQU and Path Compression

Assume we have eight sets, represented by integers 1 through 8, that start off as completely disjoint sets. Draw the WQU Tree after the series of `union()` and `find()` operations with path compression. Write down the result of `find()` operations. Break ties by choosing the smaller integer to be the root.

```plaintext
union(2, 3);
union(1, 6);
union(5, 7);
union(8, 4);
union(7, 2);
find(3);
union(6, 4);
union(6, 3);
find(7);
find(8);
```
2 Is This a BST?

The following method `isBSTBad` is supposed check if a given binary tree is a BST, though for some binary trees, it is returning the wrong answer. Think about an example of a binary tree for which `isBSTBad` fails. Then, write `isBSTGood` so that it returns the correct answer for any binary tree. The `TreeNode` class is defined as follows:

```java
class TreeNode {
    int val;
    TreeNode left;
    TreeNode right;
}
```

Hint: You will find `Integer.MIN_VALUE` and `Integer.MAX_VALUE` helpful when writing `isBSTGood`.

```java
public static boolean isBSTBad(TreeNode T) {
    if (T == null) {
        return true;
    } else if (T.left != null && T.left.val > T.val) {
        return false;
    } else if (T.right != null && T.right.val < T.val) {
        return false;
    } else {
        return isBSTBad(T.left) && isBSTBad(T.right);
    }
}

public static boolean isBSTGood(TreeNode T) {
    return isBSTHelper();
}

public static boolean isBSTHelper() {
    return true;
}
```
3 2-3 Trees and LLRB’s

3.1 Draw what the following 2-3 tree would look like after inserting 18, 38, 12, and 13.

```
   8
  /   \
4 6   14 16
 /     /   \
3 7   10  15
```

3.2 Now, convert the resulting 2-3 tree to a left-leaning red-black tree.

3.3 Extra: If a 2-3 tree has depth $H$ (that is, the leaves are at distance $H$ from the root), what is the maximum number of comparisons done in the corresponding red-black tree to find whether a certain key is present in the tree?
4 Hashing

4.1 Here are three potential implementations of the Integer’s hashCode() function. Categorize each as either a valid or an invalid hash function. If it is invalid, explain why. If it is valid, point out a flaw or disadvantage.

```java
public int hashCode() {
    return -1;
}
```

```java
public int hashCode() {
    return intValue() * intValue();
}
```

```java
public int hashCode() {
    return super.hashCode();
}
```

4.2 Extra, but highly recommended: For each of the following questions, answer Always, Sometimes, or Never.

(a) When you modify a key that has been inserted into a HashMap will you be able to retrieve that entry again? Explain.

(b) When you modify a value that has been inserted into a HashMap will you be able to retrieve that entry again? Explain.