1.1 Write the graph above as an adjacency matrix, then as an adjacency list. What would be different if the graph were undirected instead?

1.2 Give the DFS preorder, DFS postorder, and BFS order of the graph traversals starting from vertex $A$. Break ties alphabetically.

1.3 Give a valid topological sort of the graph. (Hint: Consider the reverse postorder of the whole graph.)
Graphs

Graph Algorithm Design

2.1 An undirected graph is said to be bipartite if all of its vertices can be divided into two disjoint sets \( U \) and \( V \) such that every edge connects an item in \( U \) to an item in \( V \). For example below, the graph on the left is bipartite, whereas on the graph on the right is not. Provide an algorithm which determines whether or not a graph is bipartite. What is the runtime of your algorithm?

![Bipartite Graphs](image)

2.2 Provide an algorithm that finds the shortest cycle (in terms of the number of edges used) in a directed graph in \( O(EV) \) time and \( O(E) \) space, assuming \( E > V \).

2.3 Consider the following implementation of DFS, which contains a crucial error:

- create the fringe, which is an empty Stack
- push the start vertex onto the fringe and mark it
- while the fringe is not empty:
  - pop a vertex off the fringe and visit it
  - for each neighbor of the vertex:
    - if neighbor not marked:
      - push neighbor onto the fringe
      - mark neighbor

Give an example of a graph where this algorithm may not traverse in DFS order.