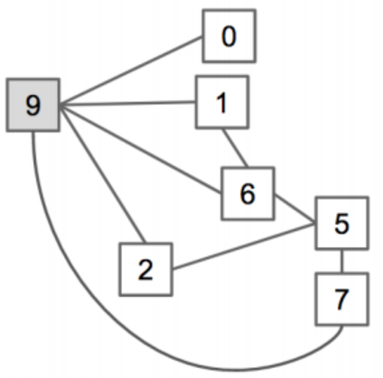
CS 61B
Spring 2018

Graphs

Exam Prep 11: April 2, 2018

1 Warmup with DFS and BFS



(a) For the graph above, give the depth first search preorder traversal starting from vertex 9, assuming that we break ties by visiting smaller numbers first.
9, 0, 1, 6, 5, 2, 7

(b) For the graph above, give the depth first search postorder traversal starting from vertex 9, assuming that we break ties by visiting smaller numbers first.
0, 2, 7, 5, 6, 1, 9

(c) For the graph above, give the breadth first search traversal starting from vertex 9, assuming that we break ties by visiting smaller numbers first.
9, 0, 1, 2, 6, 7, 5

2 Cycle Detection

Given an undirected graph, provide an algorithm that returns true if a cycle exists in the graph, and false otherwise. Also provide a Θ bound for the worst case runtime of your algorithm. You may use either an adjacency list or an adjacency matrix to represent your graph.

We do a depth first search traversal through the graph. While we recurse, if we visit a node that we visited already, that indicates a cycle. Assuming integer labels, we can use something like a visited boolean array to keep track of the elements that we've seen, and while looking through a node's neighbors, if visited gives true, then that indicates a cycle. Note that this algorithm differs slightly if the graph is directed. If the graph is directed, then there is a cycle if a node has already been visited *and it is still in the recursive call stack (i.e. still in the fringe and not popped off yet)*. Assuming an adjacency list implementation, the runtime of this is equivalent to the runtime of depth first search, which is $\Theta(|V| + |E|)$, where $|V|$ is the number of vertices and $|E|$ is the number of edges.

3 Compiler Dependencies

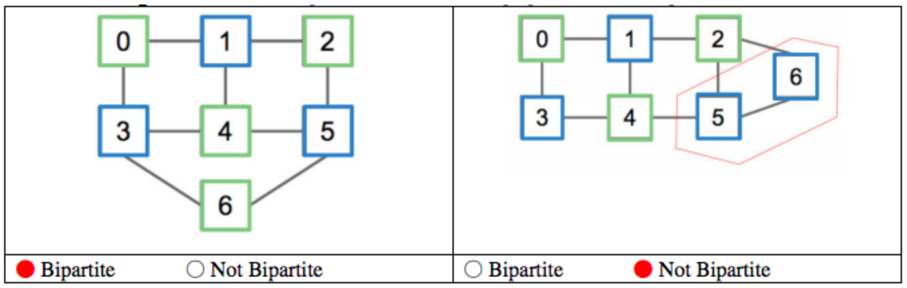
Come up with an efficient algorithm for the following problem: you are given a list of java files, some of which are dependent on one another. For example, `TestRODI.java` from the midterm was dependent on the `ReverseOddDigitIterator` class because it used that object's methods in the test. Therefore, if you want to compile `TestRODI.java`, you have to compile `ReverseOddDigitIterator.java` first. Given the list of java files, your job is to determine the ordering in which to compile the files so that all dependencies are compiled first.

Hint: Think about the different graph traversals that you learned in lecture.

We convert this information into a directed graph. The vertices will be the files, and an edge from file i to file j means that j is dependent on file i and that i needs to be compiled before j. Thus, we first construct the graph by looking through the files, and finding dependencies. The ordering of the files will be a given by a topological sort of the graph. Thus, run depth first search of the graph to get the postorder traversal of it. Return the reverse of the postorder traversal. The runtime of this function, ignoring the scanning of the files, the runtime will be $\Theta(|V| + |E|)$, where $|V|$ is the number of files and $|E|$ is the total number of dependencies.

4 Bipartite Graphs

(a) Suppose we want to color every vertex of a graph either blue or green such that no vertex touches anther vertex of the same color. This is possible for some graphs but not others. A graph where a valid coloring exists is called bipartite. Which of the graphs below are bipartite?



(b) Now fill in the method `twocolor` below such that a correct assignment to the blue vertices is printed out when the code runs, or if no such assignment is possible, an exception is thrown. Write only one statement per line. Please use the provided Graph API given to you.

```
public class Graph {
    public Graph(int V):                // Create empty graph with v vertices
    public void addEdge(int v, int w):  // add an edge v-w
    Iterable<Integer> adj(int v):       // vertices adjacent to v
    int V():                            // number of vertices
    int E():                            // number of edges
    ...
}
```

```
HashSet<Integer> blue = new HashSet<Integer>();
HashSet<Integer> green = new HashSet<Integer>();
twocolor(G, 0, blue, green);
System.out.println(Blue vertices are:  + blue.toString());

public static void twocolor(Graph G, int v, Set<Integer> a, Set<Integer> b){
    a.add(v);
    for (int u : G.adj(v)) {
        if (a.contains(u)) {
            throw new IllegalArgumentException("graph is not bipartite"); }
        if (!b.contains(u)) {
            twocolor(G, u, b, a);
        }
    }
}

```